CHAPTER - 2

MAGNETIC MATERIALS

* Terminology:-

① Magnetic flux density or magnetic Induction:- ($\vec{B}$)

Magnetic ~~Induction~~ flux is defined as the number of magnetic lines Passing normally through the surface. The magnetic flux per unit area of a section normal to the direction of the flux is called magnetic Induction.

The CGS unit of magnetic induction is Gauss
SI unit of magnetic Induction is Tesla

1 Gauss = 10^4 weber/m^2
1 Tesla = 1 weber/m^2

② Magnetising field or magnetic field Strength ($\vec{H}$)

(It is defined as the field in which a substance is Placed)

~~magnetic~~ ⓐ The field in which a substance is Placed is known as magnetic field.

ⓑ The magnetitude of the magnetic field is usually given in units of Gauss (G) or Tesla (T)

③ Intensity of Magnetization (I):-

It is defined as the magnetic moment per unit volume

$$M \text{ or } I = \frac{m}{V} = \frac{\text{Magnetic moment}}{\text{volume}}$$

It measures the degree of magnetization of a magnetized substance

4) Magnetic Susceptibility :- (χ)

It is defined as the ratio of the Intensity of magnetization Produced in the Substance to the magnetizing field.

$$\chi_m = \frac{I}{H} \text{ or } \frac{M}{H}$$

5) Magnetic Permeability :-

It is defined as the ratio of the magnetic Induction (B) in the medium to the magnetic field Strength (H)

$$\mu_a = \frac{B}{H} \quad \text{---} (1)$$

Where μ_a is the absolute Permeability of a medium. It is measure of the degree to which lines of force can penetrate or Permeate the medium.

$$\mu_a = \mu_0 \mu_r \quad \text{---} (2)$$

where μ_0 is absolute Permeability of free space.
μ_r is relative permeability of medium

$$\mu_a = \mu_0 \mu_r = \frac{B}{H} \quad \text{---} (3)$$

For free Space $\mu_r = 1$

$$\mu_0 = \frac{B}{H} \Rightarrow \boxed{B = \mu_0 H} \quad \text{---} (4)$$

6) Magnetic dipole moment ($\vec{M}$) :-

A System of two equal and opposite magnetic Poles Separated by Some distance is Said to Constitute a magnetic dipole.

The dipole moment of magnetic dipole is equal to Product of Pole Strength of any Pole and the distance between the Poles

$$\vec{M} = m \vec{l}$$

where m = Pole Strength of each Pole
l = distance between Pole

Every current carrying loop also behaves as magnetic dipole and its dipole moment is

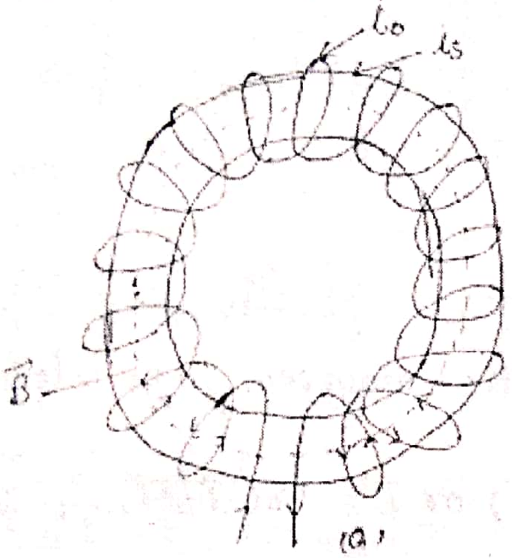
$$\vec{M} = NIA\hat{n}$$

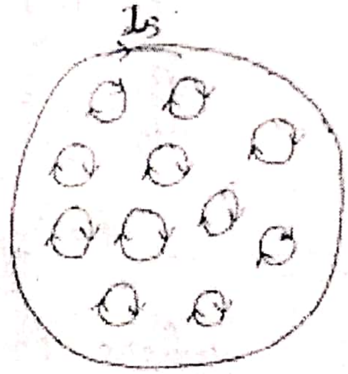
where N = number of turns in the loop
I = current through each turn
A = Area of each turn

$\hat{n}$ = unit vector normal to plane of loop and its direction found by applying right hand thumb rule

* Relation between B, H, I :-

when a magnetic material is placed in an external magnetic field, it gets magnetised. The ability of the external magnetising field to magnetise the material is represented by a vector, $\vec{H}$.





Consider a toroidal winding of N turns

Let i_0 = real current passed by winding

i_s = surface current

Let A = Area of cross section of ring

l = Length of the ring

Thus magnetisation vector is given by

$$I \text{ or } M = \frac{\text{Induced dipole moment}}{\text{volume}} = \frac{A\, i_s}{A\, l} = \frac{i_s}{l} \quad \text{--- (1)}$$

The magnetising field B_M due to the magnetisation of the material, is

$$B_M = \mu_0 I \quad \text{--- (2)}$$

The magnetising field produced due to free current in N turns is given by

$$B_0 = \mu_0 \left(\frac{N i_0}{l} \right) \quad \text{--- (3)}$$

Thus the net flux density at any point is

$$B = B_0 + B_M \quad \text{--- (4)}$$

Using eq (2) and (3) in (4)

$$B = \mu_0 I + \mu_0 \frac{N i_0}{l} \quad \text{—— (5)}$$

Using eq (1) in (5)

$$B = \mu_0 \frac{i_s}{l} + \mu_0 \frac{N i_0}{l}$$

$$= \mu_0 \left(\frac{i_s}{l} + \frac{N i_0}{l} \right)$$

$$= \mu_0 \left(I + N\frac{i_0}{l} \right)$$

$$\frac{B}{\mu_0} - I = \frac{N i_0}{l} \quad \text{—— (6)}$$

$$H = \frac{N i_0}{l} \quad \text{—— (7)}$$

where $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{I}$ (8) is called magnetic field intensity or magnetic field

Hence $\vec{B} = \mu_0(\vec{H} + \vec{I})$ or $\vec{B} = \mu_0(\vec{H} + \vec{M})$ —— 9(a)

In C.G.S system

$\vec{B} = (\vec{H} + 4\pi \vec{I})$ or $\vec{B} = (\vec{H} + 4\pi \vec{M})$ —— 9(b)

If the magnetic material is removed from the ring leaving a vacuum then $I = 0$ therefore

$$\boxed{\vec{B} = \mu_0 \vec{H}} \quad \text{—— (10)}$$

(2) Relation between M and χ:→

If magnetic material is introduced in a uniform magnetic field then total flux density is

$$B = \mu_0 H + \mu_0 I \quad \text{—— (1)}$$

as $\mu_a = \mu_0 \mu_r = \frac{B}{H}$ —— (2)

$$\mu_a = \frac{\mu_0 H + \mu_0 I}{H}$$

$$\mu_a = \mu_0 + \mu_0 \frac{I}{H} \quad \text{—— (3)}$$

$$\mu_a = \mu_0\left(1 + \frac{I}{H}\right)$$

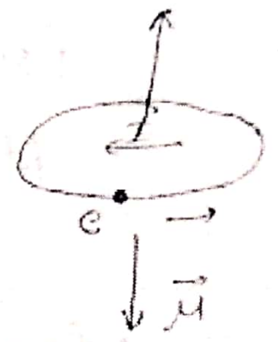
$$\mu_r = \frac{\mu_a}{\mu_0} = (1 + \chi_m) \quad \text{—— (4)}$$

$$\boxed{\mu_r = 1 + \chi_m} \quad \text{—— (5)}$$

Magnetic Dipole of Atom :-

In an atom electron revolving around the nucleus in a closed orbit. Since electron is a charged Particle, so its orbit around the nucleus is equivalent to a current loop. Hence it behaves as a magnetic dipole.

Let us assume that orbit of electron is circular



If r = radius of orbit

e = charge on electron

T = Time Period of orbital motion.

$$I = \frac{e}{T} \quad \text{—— (1)}$$

v = velocity of the electron

$$= \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v} \quad \text{—— (2)}$$

Substitute in eq (1)

$$I = \frac{ev}{2\pi r} \quad \text{—— (3)}$$

The magnetic dipole moment

$$|\vec{\mu}_e| = IA \qquad \text{where } A = \text{area of orbit}$$

$$= \left(\frac{ev}{2\pi r}\right)(\pi r^2)$$

$$\mu_e = \frac{evr}{2} \quad \text{—— (4)}$$

Multiply and divide R.H.S of eq (4) by m

$$\mu_e = \left(\frac{e}{2m}\right) mvr$$

$$= \left(\frac{e}{2m}\right) L \quad \text{—— (5)}$$

where L = angular moment of electron

In vector notation

$$\vec{\mu}_e = \left(-\frac{e}{2m}\right)\vec{L} \quad \text{—— (6)}$$

Since electron is negatively charged thus $\vec{\mu}$ and $\vec{L}$ are oppositely directed

(2) Magnetic materials:-

Magnetic materials are the substances, which when placed in external magnetic field are attracted or repelled by magnetic field. These can be broadly categorised as

(1) Diamagnetic (2) Paramagnetic (3) Ferromagnetic

(1) Diamagnetic materials:-

"The materials which when placed in external magnetic field are weakly magnetised in a direction opposite to the applied field are called diamagnetic materials."

e.g. → Hydrogen, air, water, gold, Silver, bismuth etc.

The important Properties of diamagnetic materials.

(a) These materials are repelled by magnetic field. If a diamagnetic material is placed in a region where some external magnetic field exists, then the magnetic field lines do not prefer to pass through the materials. as shown in fig (i)

(b) When diamagnetic materials placed in non uniform field it tends to move from stronger to weaker parts of the field. as shown in fig 2(ii)

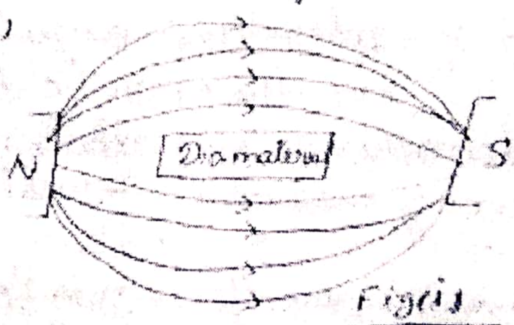


Fig (i)

(c) If a U-tube is filled with a diamagnetic liquid and external magnetic field is applied across one of the limb, then level in that limb shows depression.

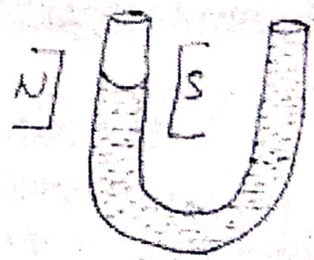
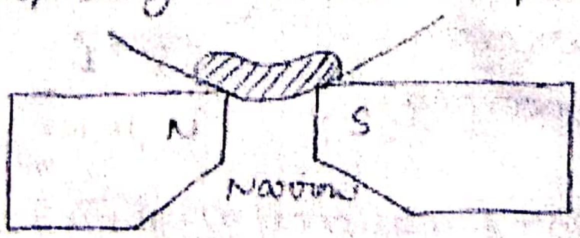


fig (iii)

(d) χ_m is slightly negative (i.e 10^{-6})
$\mu_r < 1$ (Slightly less than unity)

(e) The magnetic susceptibility is almost Independent of the temperature



From Bohr's model of atom, the angular momentum of orbiting electron is equal to an integral multiple of $\hbar$.

$$L = n\hbar \quad \text{—(7)} \quad \text{where } n = 1,2,3,4,\ldots$$

$$\hbar = \frac{h}{2\pi}$$

Using eq (7) in (6)

$$|\vec{\mu}_l| = \frac{ne\hbar}{2m} = n\left(\frac{e\hbar}{2m}\right)$$

$$= n|\vec{\mu}_B|$$

where $\vec{\mu}_B = \frac{e\hbar}{2m} = 9.274 \times 10^{-24}\, Am^2$

$|\vec{\mu}_B|$ is called Bohr magneton. Thus no electron can have magnetic moment less than μ_B. Infact orbital magnetic moment of an electron is always quantised, the minimum quantum being Bohr's magneton's

<u>Spin angular momentum</u>

Just as e^- revolving around nucleus, in the same manner, it is simultaneously revolving around its own axis. Thus it possesses spin angular momentum and hence spin magnetic moment, $\vec{\mu}_s$.

$$\vec{\mu}_s = -\frac{e}{m}\vec{S}_z$$

where S_z = spin angular momentum

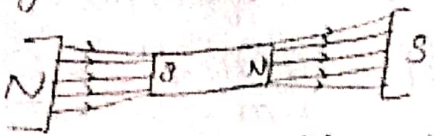
$$= \pm\frac{\hbar}{2}$$

$$\therefore \vec{\mu}_s = \pm\frac{e\hbar}{2m} = \pm\vec{\mu}_B$$

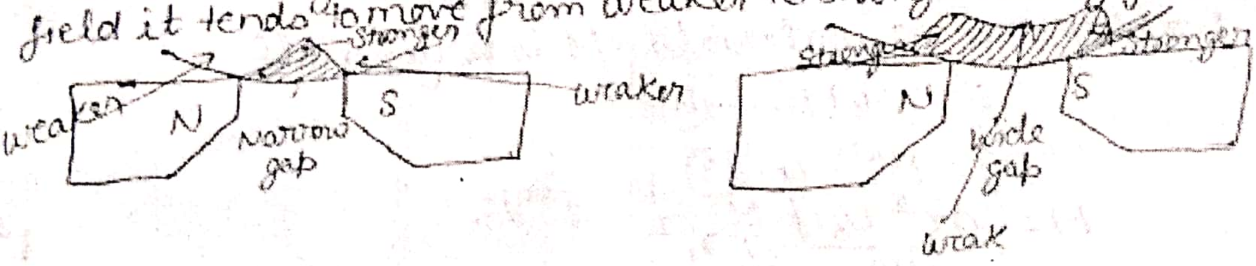
Thus net magnetic dipole moment of the atom is

$$\vec{\mu} = \vec{\mu}_l + \vec{\mu}_s$$

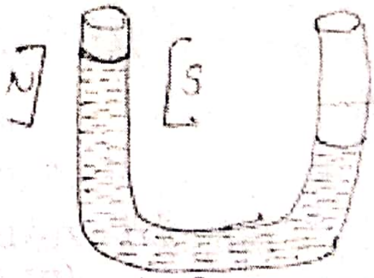
④ Paramagnetic materials:-
"The materials which when Placed in an external magnetic field are weakly magnetised in a direction to the applied field are called Paramagnetic material"
In other words "these are the materials which when Placed in external magnetic field are feebly attracts the external magnetic field"
eg:- Platinum, Aluminium, manganese, copper Sulphate, Liquid oxygen etc.
Properties of Paramagnetic materials
a) when Paramagnetic materials are Placed in uniform magnetic field then the magnetic field lines attracted by the materials



b) when a Paramagnetic material is Placed in a non-uniform magnetic field it tends to move from weaker to stronger Part of field



c) If a U-tube is filled with a Paramagnetic ~~material~~ liquid and external field is applied across one of the limbs, then level in that limb ~~s~~ level rises



d) The susceptibility of Paramagnetic material is inversely Proportional to temp. $\chi_m \propto \frac{1}{T} = \frac{C}{T}$ T = absolute temperature of Substance
χ_m is slightly Positive (10^{-3} to 10^{-5})
$\mu_r > 1$ (slightly greater than unity)

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Ferromagnetic materials:-

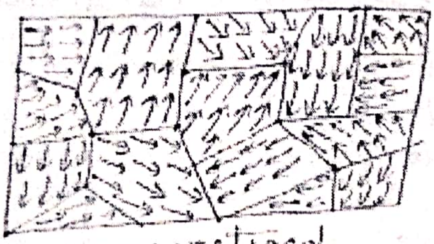
Those materials which when placed in external magnetic field are strongly magnetised in the direction of magnetic field are known as Ferromagnetic materials.

Eg:- Iron, Nickel, Cobalt, Gadolinium and their alloys.

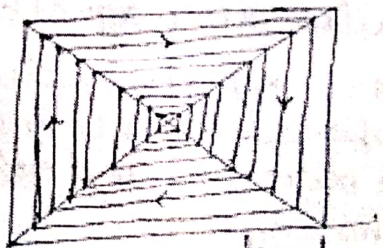
Domains theory of ferromagnetism

Ferromagnetic substances possess a magnetic moment in the absence of external magnetic field. Weiss proposed the hypothetical concept of ferromagnetic domains. He postulated that the neighbouring atoms of ferromagnetic materials due to certain mutual interactions form innumerable number of very small region called domains.

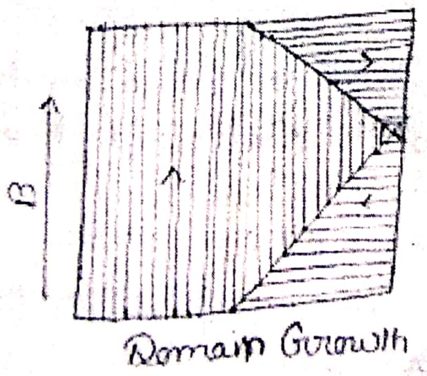
The domains are microscopic size of the order of 10^{-8} to $10^{-12}\,m^3$ and contain about 10^{17} to 10^{21} atoms. Each domain is magnetically self saturated. overall magnetisation of the specimen is given by the vector sum of magnetisations of different domains.



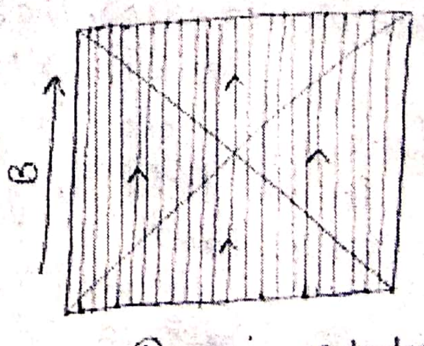
Unmagnetised State



Unmagnetised



Domain Growth



Domain rotation

Properties:-

① Magnetic susceptability χ_m (+ve and 10^6)
$\mu_r \rightarrow$ few thousands ($\times 1000$)

② In this case $B \neq \mu H$

③ with increase in temperature μ decreases (T_c = Curie temperature)
At $T = T_c$ $\quad \mu = \mu_0$ for $T > T_c$ materials becomes paramagnetic
eg:- Iron $T_c \approx 770°C$

Curie weiss law of Ferromagetism:-

Acc to weiss every molecule in a ferromagnetic material is magnetic dipole and is a source of magnetic field itself. A molecule experiences the magnetic field of the other molecules. This magnetic field is called internal field Thus it is the ~~Let H = applied magnetic field~~ internal field, which is responsible for lining up all the dipoles of a local cluster in one direction and result in the formation of ferromagnetic domains

Let H is applied magnetic field

M is magnetisation of sample

Then Acc to weiss, the Internal field H_i is given as

$$H_i = H + \gamma M \quad \text{—— (1)}$$

where γ is molecular field constant or weiss constant.

The internal field H_i of ferromagnetic material is found to obey's Curie's law

$$\chi_i = \frac{C}{T} \quad \text{—— (2)}$$

Now $$\chi_i = \frac{M}{H_i} \quad \text{—— (3)}$$

Comparing eq. (2) and (3)

$$\frac{M}{H_i} = \frac{C}{T} \quad \text{—— (4)}$$

Using eq (1) in (4)

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$$\frac{M}{H+\gamma M} = \frac{C}{T}$$

$$MT = HC + \gamma CM$$

$$M(T - \gamma C) = HC$$

$$M = \frac{HC}{T - \gamma C}$$

$$\frac{M}{H} = \frac{C}{T - \gamma C}$$

$$\chi_m = \frac{C}{T - T_c} \quad \text{———} \quad (5)$$

where $T_c = \gamma C$ is called curie temperature

For $T < T_c$, matterial behaves as ferromagnetic material

$T > T_c$, matterial becomes Paramagnetic

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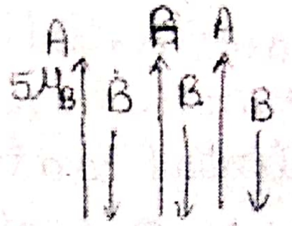
Ferrimagnetic materials :- Ferrites

In certain metallic oxide and ceramic materials magnetic moment are aligned antiparallel ~~…~~ But the magnitude of ions are different so they have net magnetization



Ferrites are generally represented by $[XO\,Fe_2O_3]$

where X = Mn, Ni, Cu, Mg, Zn, Cd, Co etc ions

$A \rightarrow 5\mu_B$
$B \rightarrow 3\mu_B$

e.g :- ① $ZnO\,Fe_2O_3 = ZnFe_2O_4$

② $FeO\,Fe_2O_3 = Fe_3O_4$

③ $CdO\,Fe_2O_3 = Cd\,Fe_2O_3$ etc

Properties :-

② $\chi = \frac{C}{T}$ at Normal temp.

$\chi = \frac{C}{T - T_c}$ at high temp

③ They have high Permeability

④ They have low Conductivity

⑤ high resistivity of range (0.1 ohm to 10 Ωm)
(For iron ferromagnetic matterial = 10^{-7} Ωm)

⑥ At high frequency they have eddy current loss are same so they are used in microwave Systems

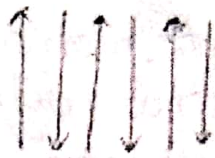
Application of ferrites :- 1) As ferrites are bad conductor of electricity, but have large Saturation magnetization, they are useful in high frequency with low eddy current Loss

2) Mn-Zn ferrites are used in low freq transformer and filters

3)

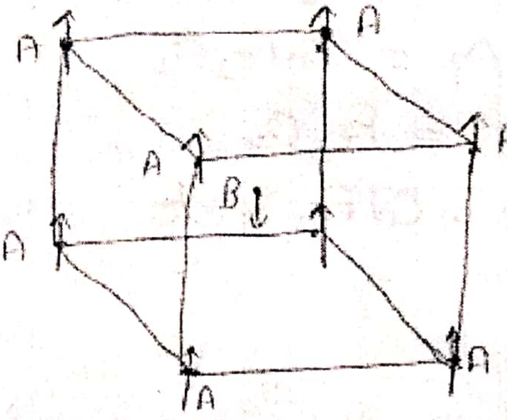
Antiferromagnetic materials→

Antiferromagnetic materials also Possess Permanent magnetic dipoles These dipoles align themself with their dir antiParallel to each other. Hence Net magnetic dipole moment of the material is always zero.



e.g.→ MnO, $FeCl_2$, FeO, NiO, CoO and Cr_2O_3 etc

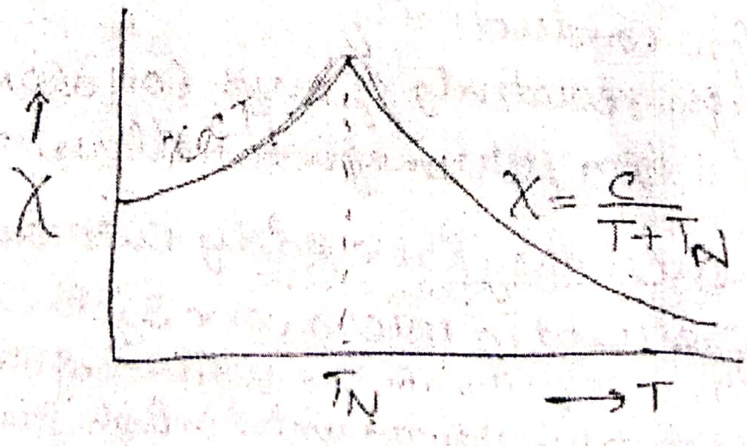
Structure→



Properties→ (1) In absence of field $\Sigma \vec{P_N} = \vec{0}$

(2) In Presence of $\vec{B}$ → small magnetisation takes Place

(3)



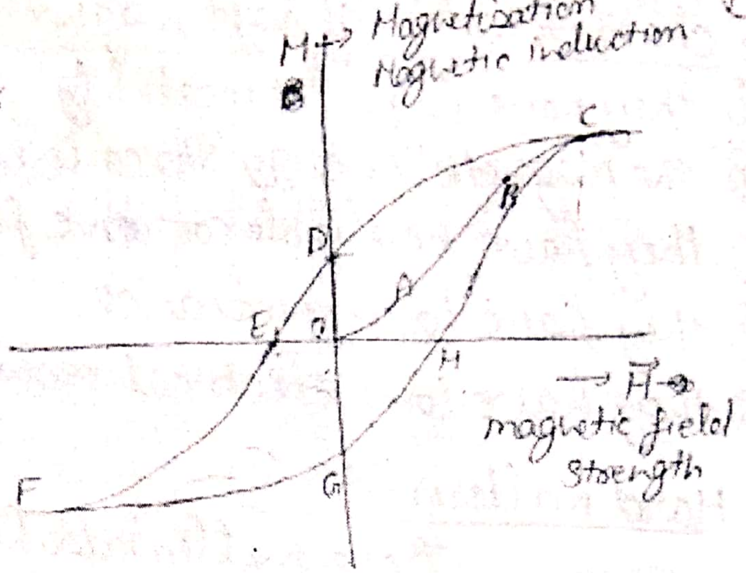
(6)
(69)

T_N → Neel temp

Hysteresis loop ↦

CD → ΔM = Retentivity or Reluctance

OE = ΔH = Coercivity



when a magnetic field of flux density B is applied to specimen change in $d|\vec{\mu}|$ dipole doing the work done

dW = change in Potential energy

$= d|\vec{\mu}|\, B \cos 0°$

$$\frac{dW}{V} = \frac{|d\vec{\mu}|}{V} B \text{ --- (1)} = dM\, B \text{ --- (2)}$$

Now we know $B = \mu_0 H$ --- (3)

$$\frac{dW}{V} = dM \mu_0 H = \mu_0 H\, dM$$

W = total work done per unit volume

$$= \oint \frac{dW}{V} = \mu_0 \oint H\, dM$$

$$= \mu_0 [\text{area of Hysteresis loop}]$$

Importance of Hysteresis loop ↦

Hysteresis loop gives heat lost per unit volume per cycle

This is called Warburg law

★ Hard & Soft magnetic matterials :-

Soft magnetic matterials ↦ These matterials are easy to magnetised to the direction of field are known as soft magnetic matterials and their coercive force is low. i.e It is easy to remove the domain walls in soft magnetic matterials

e.g. ↦ Iron-Silicon alloy, Iron-Nickel alloy.

Characteristics of soft magnetic materials

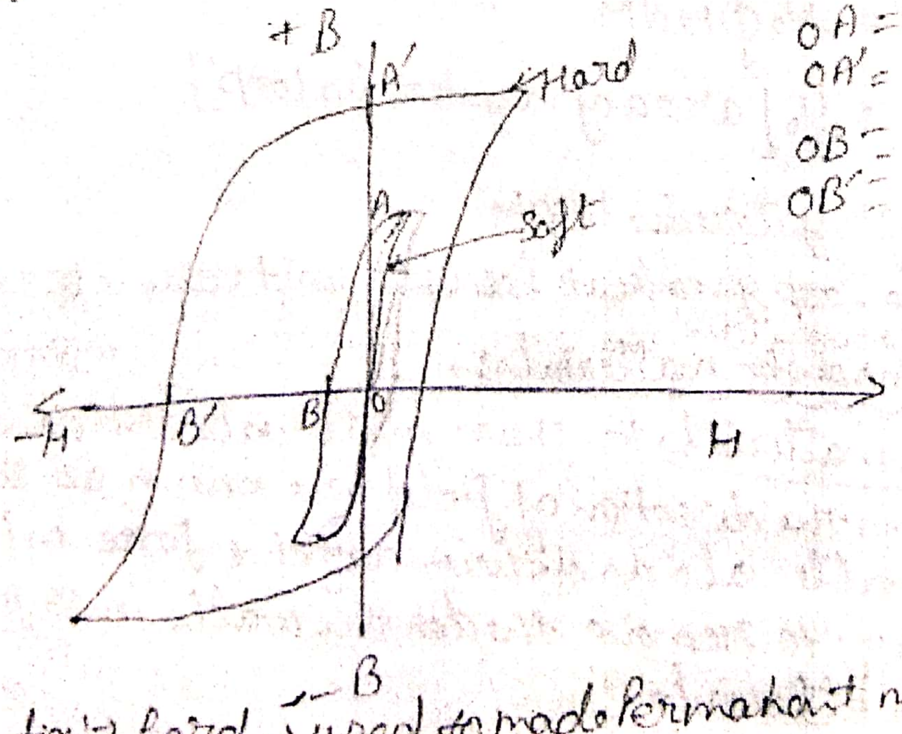
① they have high Permeability
② the magnetic energy stored is not high
③ they have negligible coercive force
④ they have low remanance
⑤ they have low electrical resistivity and low hysteresis loss

Hard matterials:-

These matterials which is difficult to move the domain walls, the corecive force is large are known as magnetically hard matterials

Characteristics of Hard magnetic matterials

① they Possess high value of energy Product (BH value)
② they have high retentivity and high coercivity
③ they have strong magnetic reluctance
④ they have hysteresis loop rectangular in shape
⑤ they have low initial Permeability and high hysteresis energy losses



OA = retentivity of soft
OA' = retentivity of h
OB = coercivity of soft
OB' = " " hard

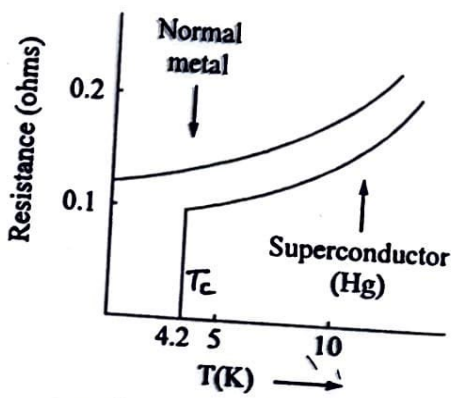
Application:- hard → used to made Permanent magnet
Soft → Transformer cores, due to less energy loss

Superconductivity

The electrical resistivity of many metals and alloys drops suddenly to zero when they are cooled to a sufficiently low temperature.

The phenomenon of disappearance of electrical resistance below a certain temperature is known as the superconductivity and specimens are called superconductor.

The resistivity of Hg vanishes completely below temp. of 4.2 K. This temp. is known as transition Temp. T_c.



The important properties of superconductors are as below :-

(*a*) The current in the superconductors persists for a very long time. This is demonstrated by placing a loop of the superconductor in a magnetic field, lowering its temperature below transition temperature T_c and then removing the field. The current which is setup is found to persist over a period longer than year without any attenuation.

(*b*) The magnetic field does not penetrate into the body of the superconductor. The property known as the *Meissner effect*, is the fundamental characterization of superconductivity. However, when the magnetic field H is greater than a critical field $H_C(T)$, the superconductor becomes a normal conductor.

(*c*) When a current through the superconductor is increased beyond a critical value $I_C(T)$, the superconductor again becomes a normal conductor, *i.e.*, the magnetic field which causes a superconductor to become normal from a superconducting state is not necessarily an external magnetic field, it may arise as a result of electric current flow in the conductor, the superconductivity may be destroyed when the current exceeds the critical value I_C, which at the surface of the write will produce a critical field H_C given by

$$I_C = 2\pi r H_C$$

This is known as Silsbee's rule.

(*d*) The specific heat of the material shows an abrupt change at $T = T_C$, jumping to a large value for $T < T_C$.

(e) T_C increases with a high power of the atomic volume and inversely as the atomic mass and is known as **isotope effect.**

(f) **Superconductivity occurs in materials having high normal resistivities.**

Magnetic Properties of Super conductors :→

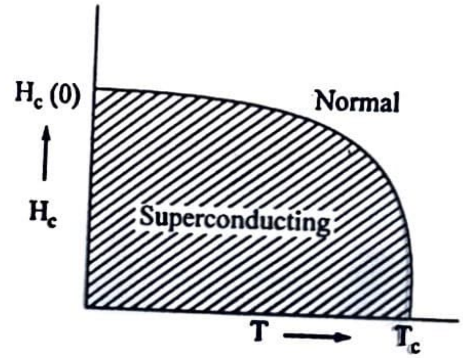
If a superconductor has the form of ring and a current is set up by electromagnetic induction, the current continue to persist for infinite time below the critical temperature. These are called persistent currents.

Also the application of a sufficiently strong magnetic field to the superconductor causes the destruction of their superconductivity. The value of magnetic field at which the superconductor changes to normal state is called critical magnetic field H_c and it is related to the temp as

$$\boxed{H_c = H_c(0)\left[1 - \frac{T^2}{T_c}\right)}$$

Here $H_c(0)$ is value of critical magnetic field at $T=0$.

T_c — is transition temp.

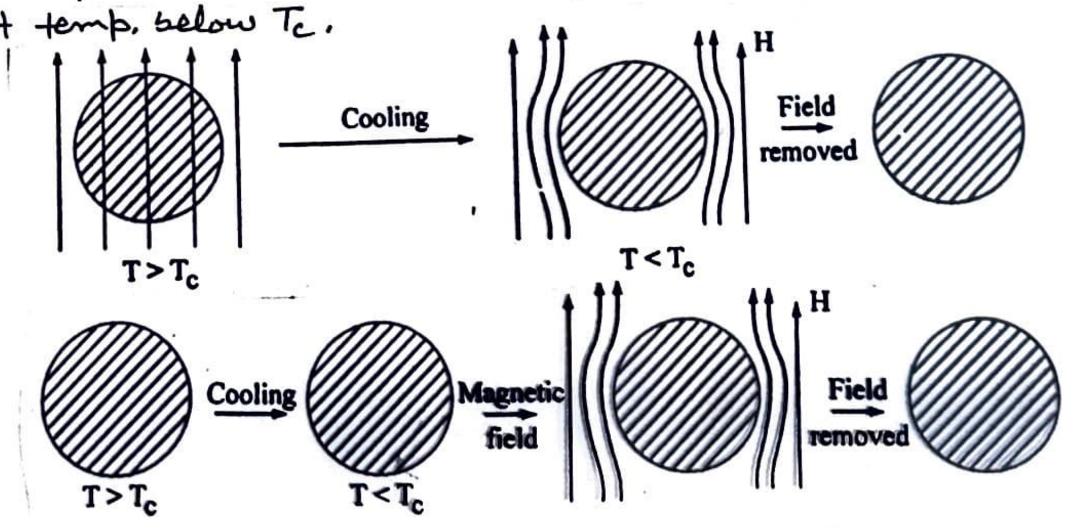


Variation of critical field H_c with temperature

The Meissner Effect :→

When a superconductor is cooled in magnetic field below the value of transition temp., then the lines of magnetic induction B are pushed out of the body of superconductor at the transition. This is known as Meissner Effect.

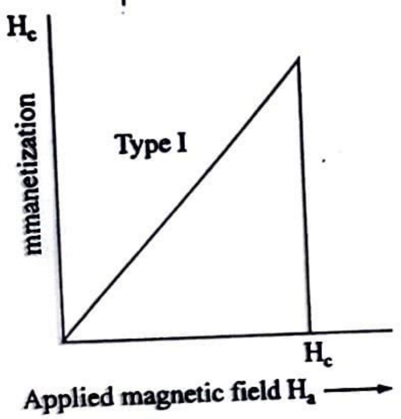
Also, if the superconductor is first cooled below temp T_c and then placed in magnetic field, the magnetic induction lines B are pushed out of the body of superconductor. So superconductor behave as perfect diamagnetic substance at temp. below T_c.

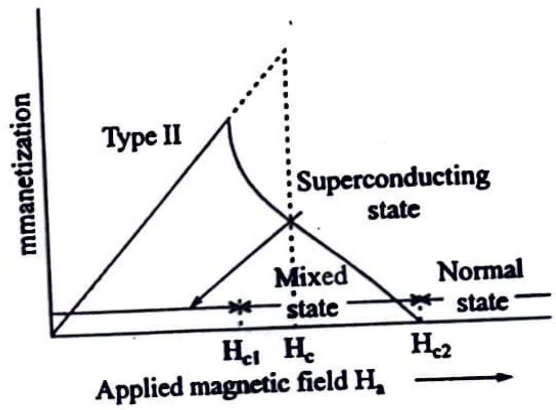


Type I and Type II Superconductors :→

Type I :→ The type I superconductors behave as completely diamagnetic at the temp. below Tc. The magnetic flux is completely excluded from the body of type I superconductor below Tc.
These are also called soft superconductor because these type I superconductor give away their superconductivity at low field strength.

Type II Superconductor :→





In type II superconductor, there are two critical magnetic fields H_{c1} and H_{c2} which are known as lower critical field and upper critical field H_{c2}.

For applied fields below H_{c1}, the specimen is diamagnetic and hence the flux is completely excluded in this range of field; H_{c1} is called the lower critical field. At H_{c1} the flux begins to penetrate the specimen, and the penetration increases until H_{c2} is reached. At H_{c2} the magnetization vanishes and the specimen becomes normal conductor; H_{c2} is called the upper critical field. Moreover, the magnetization of this group of superconductor vanishes gradually as the field is increased, rather than suddenly as for the type I superconductors. However, they are completely superconducting for all fields below H_{c2}. The superconductors of this group are called type II superconductors. They tend to be alloys or transition metals with high values of the electrical resistivity in the normal state. Type II superconductors are technically very useful materials, in contrast to type I superconductors.

Thermodynamics of Superconductor :→

To study the difference ~~between~~ in entropy S specific heat c between normal and superconducting states, consider the Gibb's free energy G. The Gibb free energy G per unit volume in magnetic field is given as

$$G = U - TS - HM + PV$$

Since there is small change in P and V, so neglecting term PV, we have

$$G = U - TS - HM \qquad \text{——— (1)}$$

where H is applied field and M is magnetisation.

The internal energy U is given by (from IInd law of thermodynamics),

$$dU = TdS + HdM \qquad \text{——— (2)}$$

HdM → is the work done on superconductor per unit volume.

Differentiating equ. (1), we have

$$dG = dU \cancel{+} TdS - SdT - MdH - HdM \qquad \text{——— (3)}$$

From (2) and (3),

$$dG = TdS + HdM - TdS - SdT - MdH - HdM$$

$$\Rightarrow \boxed{dG = -TdS - MdH} \qquad \text{——— (4)}$$

Also, at constant temperature, $dT = 0$,

$$dG = -MdH \qquad \text{——— (5)}$$

Integrating the equ. (5), from 0 to H, the change in Gibb's energy for superconducting state is,

$$\int_0^H dG = \int_0^H MdH \quad \text{or} \quad G(H) - G_S(0) = -\int_0^H MdH \qquad \text{——— (6)}$$

Now if the sample is in the normal state, then the magnetisation $M \to 0$, so the equ. (6) can be written for normal state as

$$G_N(H) - G_N(0) = 0 \Rightarrow \boxed{G_N(H) = G_N(0)} \qquad \text{——— (7)}$$

⇒ The Gibb's energy remains constant

Now, at equilibrium, the Gibb's free energies of superconding state and the normal state must be equal.

$$\Rightarrow G_N(T, H_c) = G_S(T, H_c) \qquad \text{——— (8)}$$

From equ. (6) and (8), we have

$$G_N(T, H_c) = G_S(T, H_c) = G_S(0) - \int_0^H M\,dH \quad \text{—— (9)}$$

Equs (7) and (9) gives

$$G_N(T, H_c) = G_N(0) = G_S(0) - \int_0^H M\,dH$$

$$\Rightarrow G_N(0) = G_S(0) - \int_0^H M\,dH$$

$$\Rightarrow G_N(0) - G_S(0) = -\int_0^H M\,dH$$

$$\boxed{\Delta G = -\int_0^H M\,dH} \quad \text{—— (10)}$$

Equ. (10) gives the change in Gibb's free energy when specimen changes from normal state to superconducting state.

Now, from Meissner effect, the magnetic susceptibility $\chi = -1$ and we know that for superconding state,

$$M = \chi H \quad \Rightarrow$$

$$\boxed{M = -H} \quad \text{—— (11)}$$

From (10) & (11), we have

$$\Delta G = -\int_0^H -H\,dH = \frac{H^2}{2} \quad \text{—— (12)}$$

$$\text{or } \boxed{\Delta G = \frac{H_c^2}{2}}$$

So equ. (12) gives the change in Gibb's free energy, when specimen changes from normal state to superconducting state.

Entropy :→ (S)

Entropy is given as

$$S = -\left(\frac{\partial G}{\partial T}\right)_H \quad \text{—— (13)}$$

The entropy difference between normal and superconding state is

$$\Delta S = S_N - S_S = -\left(\frac{\partial G}{\partial T}\right) = -H_c\frac{dH_c}{dT}$$

$$\Rightarrow \boxed{\Delta S = S_N - S_S = -H_c\frac{dH_c}{dT}} \quad \text{—— (14)}$$

From eqn. (14) it is clear that
$\frac{dH_c}{dT}$ is always -ve, therefore $S_N - S_s$ is always +ve.
$\Rightarrow$ superconducting state is more ordered than the normal state.

Now the latent heat of system is

$$Q = T\,\Delta S$$
$$= T(S_N - S_S)$$
$$\boxed{Q = -TH_c \frac{dH_c}{dT}} \quad \text{—— (15)}$$

Now at $T = T_c$, $H_c = 0$

$\Rightarrow$ $\boxed{S_N = S_S = 0 \text{ and } Q = 0}$ $\Rightarrow$ at transition the latent heat is zero if $H_c = 0$

If $H_c \neq 0$, then transition occur at temp $T < T_c$ and latent heat is not equal to zero.

Specific Heat :→

specific heat of a solid is given by

$$C = T\frac{dS}{dT} \quad \text{—— (16)}$$

So the specific heat difference between normal and superconducting state can be obtained as

$$C_N - C_S = T\frac{\Delta S}{\Delta T} = T\frac{d(S_N - S_S)}{dT}$$

$$\Rightarrow C_N - C_S = T\frac{d}{dT}\left(-H_c\frac{dH_c}{dT}\right)$$

$$= -T\frac{dH_c}{dT}\cdot\frac{dH_c}{dT} + T(-H_c)\frac{d^2H_c}{dT^2}$$

$$\Rightarrow \boxed{C_N - C_S = -\left[TH_c\frac{d^2H_c}{dT^2} + T\left(\frac{dH_c}{dT}\right)^2\right]} \quad \text{—— (17)}$$

At $T = T_c$, $H_c = 0$ and $\frac{dH_c}{dT}$ is -ve.

$$\therefore C_N - C_s = T_c\left(\frac{dH_c}{dT}\right)^2 = -ve \quad \text{—— (18)}$$

$\Rightarrow$ $\boxed{C_N < C_S \text{ at } T = T_c}$

But as temp. decreases below T_c, $C_N - C_S$ is +ve.

The London Equation :→

This theory is based on the ideas of two fluid model, according to which the superconductor can be thought to be composed of both normal and superconducting electrons.

Let n_n, n_s are density of normal and superconducting electrons and v_n and v_s are their velocities respectively.

Let n_0 be the no. of electron per unit volume on average.

$$\Rightarrow \boxed{n_0 = n_n + n_s} \qquad \text{———— (1)}$$

The assumption of zero resistivity in superconductivity leads to the acceleration equation

$$m\frac{dv}{dt} = -eE \quad \Rightarrow \quad \frac{dv}{dt} = -\frac{eE}{m}$$

Also the current density j, number of electrons per unit volume, is

$$j = -nev$$

Therefore, we have

$$\frac{dj}{dt} = \frac{ne^2}{m}E \qquad \text{———— (2)}$$

It must be mentioned that here only the superconducting electrons are under consideration and not all the electrons : a superconductor can be supposed as composed of both normal and superconducting electrons.

The normal electrons behave like electrons in a non-conductor, and thus are of no interest to us. Further, the superconducting electrons are being assumed to respond to electric field just as free electrons do. Now taking the curl of both sides of eqn. (2) we have, with

$$\nabla \times E = \frac{\partial B}{\partial t} \quad \text{or} \quad \nabla \times \left[\frac{m}{ne^2}\frac{dj}{dt}\right] = -\frac{\partial B}{\partial t}$$

or

$$\nabla \times \left[\frac{Kdj}{dt}\right] = -\frac{\partial B}{\partial t} \qquad \text{———— (3)}$$

with

$$K = \frac{m}{ne^2}$$

Also $\quad \nabla \times B = \mu_0 j$

$$\therefore \quad \nabla \times \left(K\frac{dj}{dt}\right) = \nabla \times \left(\frac{K}{\mu_0}\frac{d}{dt}(\nabla \times B)\right) = \frac{1}{\mu_0}\nabla \times [\nabla \times (K\dot{B})] \qquad \text{———— (4)}$$

where

$$\dot{B} = \frac{d}{dt}B$$

or $$\frac{1}{\mu_0}\nabla \times [\nabla \times (K\dot{B})] = -\frac{\partial B}{\partial t}$$

Also $\nabla \times \nabla \times (K\dot{B}) = \nabla\, \nabla \,.\, (K\dot{B}) - \nabla^2 (K\dot{B})$

Since $\nabla .\, B = 0$

$\therefore$ we have

$$\frac{\partial B}{\partial t} = \frac{K}{\mu_0}\nabla^2 \frac{\partial B}{\partial t} \qquad (5)$$

Integration of above equation w.r.t. time gives

$$\frac{K}{\mu_0}\nabla^2 (B - B_0) = B - B_0 \qquad (6)$$

where B_0 is the constant of integration and denotes the field at time $t = 0$. Here the currents are considered as the only internal source of magnetic field and in this discussion no magnetization as such has been introduced.

Let us now pay attention to the fact that the equation (6) admits the particular solution $B = B_0$, where B_0 is any arbitrary field existing at $t = 0$ whereas Meissner effect tells us that we cannot have frozen in fields. So according to F. and H. London, we should eliminate B_0. This we do by abandoning the acceleration equation and taking instead

$$\nabla \times \left(\frac{mj}{ne^2}\right) = -B \qquad (7)$$

as the fundamental equation in a superconductor. Therefore, we obtain

$$\frac{K}{\mu_0}\nabla^2 B \equiv B \qquad (8)$$

this equation does not admit a constant field as a solution.

Also $\quad \nabla \times A = B$

where A is any vector field.

$$\therefore \qquad j = -\frac{ne^2}{m}A \qquad (9)$$

This is the London equation. Sometimes this equation together with eqn. (2) are called the London equations.

Using Maxwell's equation

$$\nabla \times B = \mu_0 j \qquad (10)$$

or $$\nabla \times \nabla \times B = \mu_0 \nabla \times j$$

or $$-\nabla^2 B = \mu_0 \nabla \times j \qquad (11)$$

Then, using the London equation (9), we get

$$\nabla^2 B = \frac{ne^2\mu_0}{m}\nabla \times A = \frac{ne^2\mu_0}{m}B \qquad (12)$$

or $$\nabla^2 B = \frac{1}{\Lambda^2}B \qquad (13)$$

where $\Lambda^2 = \dfrac{m}{ne^2\mu_0}$ with Λ is known as **London penetration depth**.

BCS Theory :→

These are two types of interactions in a lattice,
i repulsive electron - electron interaction.
ii interaction between the electrons and lattice ions.
The resultant of above two interaction which results in a weak attractive interaction. In this way electrons in superconductors can lower their energies.

Now consider lattice interaction. In lattice there are regions which are surrounded by +ve ions. So any electron tends to pull itself toward the +ve ions. It is a region of enhanced +ve charge density. Any other electron also will be drawn toward this region and it will look as if it was attracted towards by the first electron.

Bardeen, cooper and Schrieffer (BCS) studies this and concluded that there are attracting electron in ground state. So the ground state of an assembly of mutually attracting electron is separated by an energy gap from the lowest excited state of ~~an assembly~~ the energy spectrum.
This attractive interaction forms two electrons singlet bound states. These are called Cooper pairs.
The electrons of cooper pair have equal and opposite momenta and spin.

The wavefunction of a Cooper pair extends over a large volume and overlaps the wavefunctions of other pairs. This overlap gives rise to strong correlations among the motion of all pairs. Hence the superconducting state is a correlective state in which all the conduction electrons act co-operatively.